Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

MAT1348 3X - Practice Final Exam, Instructor: Guy Beaulieu

Summer 2016.

Family Name:____________________

First Name:____________________

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to immediately leave the exam and academic fraud allegations will be filed, which may result in you obtaining a 0 (zero) for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement: Signature:____________________
- Only the faculty-approved calculators (TI-30X, TI-34X, Casio FX-260X and Casio FX-300X) are allowed. All others will be confiscated.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- If you tear off any blank pages, they must still be handed in.
- Where it is possible to check your work, do so. Good luck!

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1. (a) [**3 points**] Construct the truth table for the following compound proposition

$$((\neg p \vee q) \rightarrow (\neg q \rightarrow p)).$$

(b) [**2 points**] Is the formula in part (a) a tautology, a contradiction, or a contingency? Justify.

(c) [**2 points**] Give a compound proposition in Disjunctive Normal Form (DNF) which is logically equivalent to Φ whose truth table is given below:

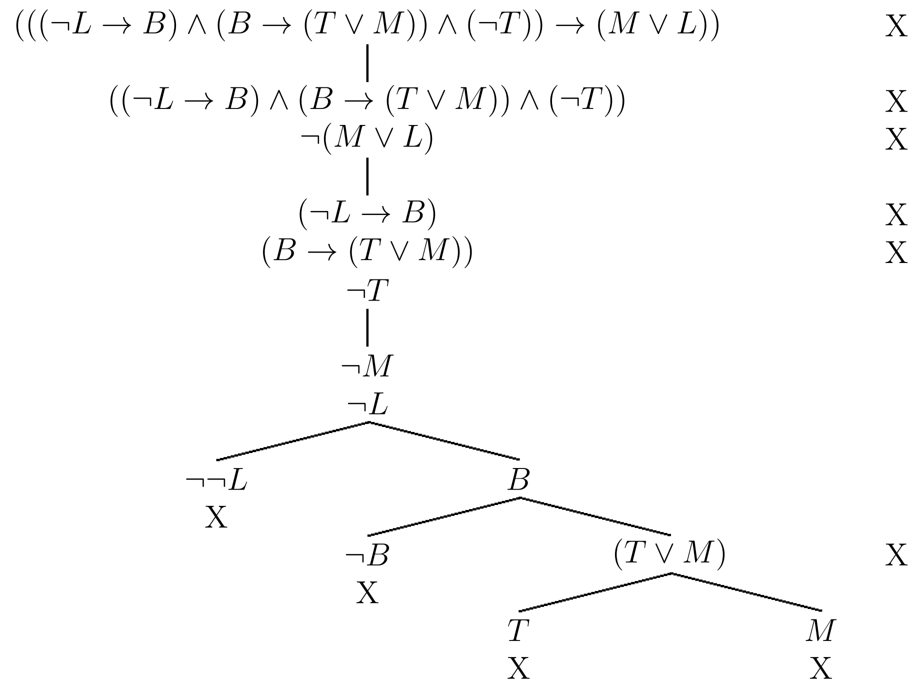
p	q	r	Φ
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

2. **[5 points]** Consider the following argument:

$$\begin{array}{c} ((p \vee \neg q) \rightarrow r) \\ (\neg r \vee p) \\ (p \rightarrow \neg q) \\ \hline r \end{array}$$

Is the argument valid? If it is not valid, give a counter-example. Justify your reasoning.

3. Consider the following truth tree:



Circle all the correct statements below which are true.

(A) The argument $\dfrac{\begin{matrix}(\neg L \to B)\\ (B \to (T \vee M))\\ \neg T\end{matrix}}{(M \vee L)}$ is valid.

(B) The compound proposition $(((\neg L \to B) \wedge (B \to (T \vee M)) \wedge \neg T) \to (M \vee L))$ is a tautology.

(C) The compound formulas $(\neg L \to B)$, $(B \to (T \vee M))$, $\neg T$, and $(M \vee L)$ are logically equivalent.

(D) The argument $\dfrac{\begin{matrix}(\neg L \to B)\\ (B \to (T \vee M))\\ \neg T\end{matrix}}{\neg(M \vee L)}$ is valid.

(E) The compound proposition $\neg(((\neg L \to B) \wedge (B \to (T \vee M)) \wedge \neg T) \to (M \vee L))$ is a tautology.

(F) The compound formulas $((\neg L \to B) \wedge (B \to (T \vee M)) \wedge \neg T)$ and $\neg(M \vee L)$ are logically equivalent.

4. For the following question, you do not need to justify your answers.
Consider the set $A = \{a, b, c, d, e\}$, and consider the relation in A given by

$$R = \{(a, a), (a, b), (a, c), (a, d), (a, e), (b, e)\}$$

(a) [**2 points**] Is R symmetric?

(b) [**2 points**] Is R reflexive?

(c) [**2 points**] Is R transitive?

(d) [**2 points**] Is R antisymmetric?

(e) [**2 points**] Give an example of a relation in A which is reflexive and transitive but which is not symmetric.

5. (a) [**4 points**] Show that

$$(B - C) \cup (C - B) = (B \cup C) - (B \cap C)$$

for all sets B and C.

(b) [**3 points**] Give an example of sets, B and C, which show the equality

$$(B - C) \cup (C - B) = B \cup C$$

is false.

6. Consider the set of integers $\mathbb{Z}$ and consider the relation

$$R = \{(n, m) \mid n - m \text{ is divisible by } 6\}$$

(i.e. R is the *congruence modulo 6 relation*, $(n, m) \in R$ iff $n \equiv m (\text{mod } 6)$).

(a) [**5 points**] Show that R is an equivalence relation on $\mathbb{Z}$.

(b) [**2 points**] What is the equivalence class of 3 in R?

(c) [**2 points**] How many distinct equivalence classes in R are there?

7. **[6 points]** Use the principle of mathematical induction to prove that

$$1 \cdot 2 + 2 \cdot 3 + \ldots + n(n+1) = n(n+1)(n+2)/3$$

where n is any positive integer.

8. To access a network, a user must choose a password. The chosen password must satisfy the following criteria:

   - the password must be between 3 and 5 characters in length.
   - the only allowed characters are the lowercase letters a, b, c, d and e and the uppercase letters U, V, W, X, Y and Z.

   (a) [**3 points**] How many valid passwords are there?

   (b) [**5 points**] Suppose that in addition, the password must contain at least one lowercase and one uppercase letter. How many passwords can now be made using this extra condition in addition to the initial ones?

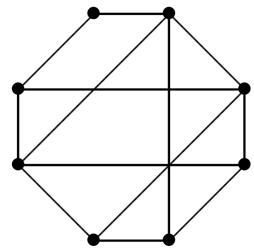
9. In a bakery we find 10 different types of doughnuts.

   (a) [**3 points**] In how many ways can we choose a doughnut to give to six children, if no child can receive the same type of doughnut?

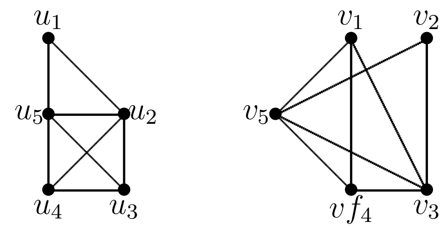
   (b) [**3 points**] In how many ways can the doughnuts be chosen for the six children if they can receive the same type of doughnut?

   (c) [**4 points**] In how many ways can we choose 6 doughnuts, knowing that there must be at least two doughnuts of different type chosen?

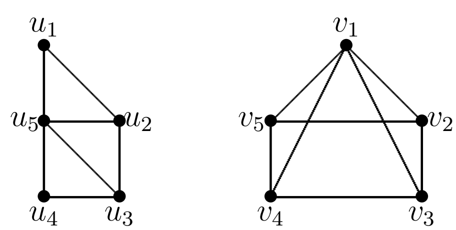
10. [**3 points**] Is the following graph bipartite? Justify your answer.



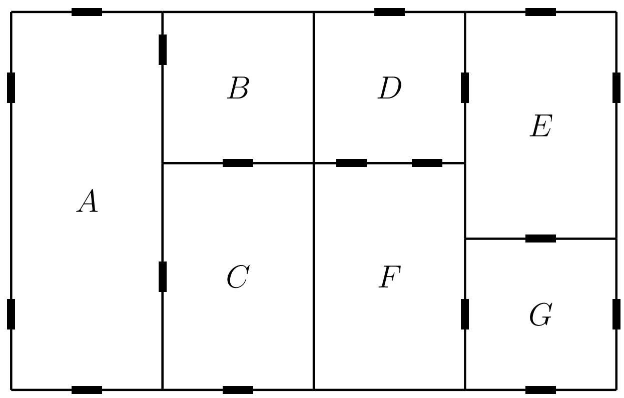
11. (a) [**3 points**] Are the following graphs isomorphic? If yes, give the isomorphism. If not, explain why.



(b) [**3 points**] Are the following graphes isomorphic? If yes, give the isomorphism. If not, explain why.



12. [**3 points**] The following diagram represents the floor plan of a house, can we visit all the rooms in the house by passing through each of the doors only once and finishing in the same room that we started in? If so, give the circuit. If not, can it be done if we finish our visit in a different room than the one we started in? If so, give the path. If not, explain why.



13. True or False? You do not have to justify your answer.

(a) [**2 points**] For all natural numbers n and k with $0 < k < n$ we have that

$$\binom{n}{k} < \binom{n}{k+1}.$$

(b) [**2 points**] There exists a simple graphe with exactly 7 vertices, all of which of degree 3.

(c) [**2 points**] An equivalence relation is never antisymmetric.

(d) [**2 points**] If a function $f : \mathbb{N} \to \mathbb{N}$ is surjective, then it is also injective.

(e) [**2 points**] There are exaclty 24 bijections from the set $\{a, b, c, d\}$ into itself.